

## 10-4 Practice

## Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

1.  $\log 101$

2.  $\log 2.2$

3.  $\log 0.05$

Use the formula  $\text{pH} = -\log[H^+]$  to find the pH of each substance given its concentration of hydrogen ions.

4. milk:  $[H^+] = 2.51 \times 10^{-7}$  mole per liter  $\text{pH} = -\log(2.51 \times 10^{-7}) = 6.6$   
 5. acid rain:  $[H^+] = 2.51 \times 10^{-6}$  mole per liter  $\approx 5.6$   
 6. black coffee:  $[H^+] = 1.0 \times 10^{-5}$  mole per liter  $\approx 5.0$   
 7. milk of magnesia:  $[H^+] = 3.16 \times 10^{-11}$  mole per liter  $\approx 10.5$

Solve each equation or inequality. Round to four decimal places.

8.  $2^x < 25$

$x \log 2 < \log 25$

$x < 4.6439$

9.  $5^a = 120$

10.  $6^z = 45.6$

11.  $9^m \geq 100$

$m \log 9 \geq \log 100$

12.  $3.5^x = 47.9$

13.  $8.2^y = 64.5$

14.  $2^b + 1 \leq 7.31$

$b \geq 2.0959$

15.  $4^{2x} = 27$

16.  $2^a - 4 = 82.1$

17.  $9^z - 2 > 38$

$z - 2 > \frac{\log 38}{\log 9}$

18.  $5^w + 3 = 17$

19.  $30^{x^2} = 50$

20.  $5^{x^2 - 3} = 72$

$x^2 - 3 > \frac{\log 72}{\log 5}$

21.  $4^{2x} = 9^x + 1$

22.  $2^n + 1 = 5^{2n} - 1$

$x^2 - 3 = \frac{\log 72}{\log 5} \quad x = 2.3785$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23.  $\log_5 12 = \frac{\log 12}{\log 5} = 1.5448$

24.  $\log_8 32 = \frac{\log 32}{\log 8} = 1.6667$

25.  $\log_{11} 9 = \frac{\log 9}{\log 11} = .9163$

26.  $\log_2 18 = \frac{\log 18}{\log 2} = 4.1699$

27.  $\log_9 6$

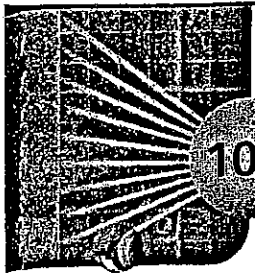
28.  $\log_7 \sqrt{8}$

29. **HORTICULTURE** Siberian irises flourish when the concentration of hydrogen ions  $[H^+]$  in the soil is not less than  $1.58 \times 10^{-8}$  mole per liter. What is the pH of the soil in which these irises will flourish?  $-\log[1.58 \times 10^{-8}] = 7.8$

30. **ACIDITY** The pH of vinegar is 2.9 and the pH of milk is 6.6. How many times greater is the hydrogen ion concentration of vinegar than of milk?

31. **BIOLOGY** There are initially 1000 bacteria in a culture. The number of bacteria doubles each hour. The number of bacteria  $N$  present after  $t$  hours is  $N = 1000(2)^t$ . How long will it take the culture to increase to 50,000 bacteria?

32. **SOUND** An equation for loudness  $L$  in decibels is given by  $L = 10 \log R$ , where  $R$  is the sound's relative intensity. An air-raid siren can reach 150 decibels and jet engine noise can reach 120 decibels. How many times greater is the relative intensity of the air-raid siren than that of the jet engine noise?



NAME \_\_\_\_\_

DATE \_\_\_\_\_

PERIOD \_\_\_\_\_

10-5

**Skills Practice****Base e and Natural Logarithms**

Use a calculator to evaluate each expression to four decimal places.

1.  $e^3 \approx 20.0855$

2.  $e^{-2}$

3.  $\ln 2 \approx 0.6931$

4.  $\ln 0.09$

Write an equivalent exponential or logarithmic equation.

5.  $e^x = 3$   $\ln 3 = x$

6.  $e^4 = 8x$

7.  $\ln 15 = x$   $e^x = 15$

8.  $\ln x \approx 0.6931$

Evaluate each expression.

9.  $e^{\ln 3}$  3

10.  $e^{\ln 2x}$

11.  $\ln e^{-2.5}$  -2.5

12.  $\ln e^y$

Solve each equation or inequality.

13.  $e^x \geq 5$   $x \geq \ln 5$

14.  $e^x < 3.2$

15.  $2e^x - 1 = 11$   $2e^x = 12$

16.  $5e^x + 3 = 18$

17.  $e^{3x} = 30$

18.  $e^{-4x} > 10$

19.  $e^{5x} + 4 > 34$

20.  $1 - 2e^{2x} = -19$

21.  $\ln 3x = 2$

22.  $\ln 8x = 3$

23.  $\ln(x - 2) = 2$

24.  $\ln(x + 3) = 1$

25.  $\ln(x + 3) = 4$

26.  $\ln x + \ln 2x = 2$

$$y = a(1-r)^t$$

$$1. \quad \frac{1}{2} = 1(1-.08)^t$$

$$\frac{1}{2} = .92^t$$

$$\ln \frac{1}{2} = t \ln .92$$

$$8.3 = t$$

$$2. \quad 3600 = 2000(1+.06)^t$$

$$1.8 = 1.06^t$$

$$\ln 1.8 = t \ln 1.06$$

$$10.09 = t$$

3.

$$P = 3.5 e^{.015 \times 30}$$

$$5.49 \text{ mill}$$

4.  $P =$

$$120 = 80 e^{.015t}$$

$$\ln 1.5 = .015t$$

$$27 = t$$

$$5. \quad 50,000 = 2000(1+.932)^t$$

$$25 = (1.932)^t$$

$$\frac{\log 25}{\log 1.932} = t$$

$$4.89 = t$$

P605

$$6. \quad \frac{1}{2} = 1e^{-k(24,360)}$$

$$\ln \frac{1}{2} = -24,360k$$

$$2.8 \times 10^{-5} = k$$

$$0.000028 = k$$

$$7. \quad 8600 = 12,500(1-.06)^t$$

$$.688 = .94^t$$

$$6.04 = t$$

$$8. \quad 250 = 100(1+r)^8$$

$$2.5 = (1+r)^8$$

$$1.1213 = 1+r$$

$$.1213 = r$$

$$12.13\%$$

~~P604~~  
~~P605~~  
~~P606~~  
~~P 253-8~~  
~~other Book~~  
 13 problems

P6004

1.

$$326 = 40 e^{K(120)}$$

$$\ln \frac{326}{40} = 120K$$

$$.0175 = K$$

$$A = Pe^{rt}$$
~~$$P = A e^{-rt}$$~~

2.

$$1500 = 500 e^{.0525t}$$

$$\ln \frac{1500}{500} = .0525t$$

$$13.3 = t$$

3.  $1250 = 850 (1 + .078)^t$

$$1.47 = 1.078^t$$

$$\frac{\log 1.47}{\log 1.078} = t$$

~~$$4.34 = t$$~~

$$5.12$$

4.  $5 = .5 (1.10)^t$

$$10 = 1.10^t$$

$$\frac{\log 10}{\log 1.10} = t$$

$$24 = t$$

25 weeks more than

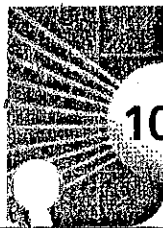
5.  $2500 = 10 (1+r)^{42}$

$$250 = (1+r)^{42}$$

$$1.14 = 1+r$$

$$.14 = r$$

$$14\%$$



NAME \_\_\_\_\_

DATE \_\_\_\_\_

PERIOD \_\_\_\_\_

**10-6****Skills Practice****Exponential Growth and Decay***Key on***Lesson 10-6**

Solve each problem.

1. **FISHING** In an over-fished area, the catch of a certain fish is decreasing at an average rate of 8% per year. If this decline persists, how long will it take for the catch to reach half of the amount before the decline?
2. **INVESTING** Alex invests \$2000 in an account that has a 6% annual rate of growth. To the nearest year, when will the investment be worth \$3600?
3. **POPULATION** A current census shows that the population of a city is 3.5 million. Using the formula  $P = ae^{rt}$ , find the expected population of the city in 30 years if the growth rate  $r$  of the population is 1.5% per year,  $a$  represents the current population in millions, and  $t$  represents the time in years.
4. **POPULATION** The population  $P$  in thousands of a city can be modeled by the equation  $P = 80e^{0.015t}$ , where  $t$  is the time in years. In how many years will the population of the city be 120,000?
5. **BACTERIA** How many days will it take a culture of bacteria to increase from 2000 to 50,000 if the growth rate per day is 93.2%?
6. **NUCLEAR POWER** The element plutonium-239 is highly radioactive. Nuclear reactors can produce and also use this element. The heat that plutonium-239 emits has helped to power equipment on the moon. If the half-life of plutonium-239 is 24,360 years, what is the value of  $k$  for this element?
7. **DEPRECIATION** A Global Positioning Satellite (GPS) system uses satellite information to locate ground position. Abu's surveying firm bought a GPS system for \$12,500. The GPS depreciated by a fixed rate of 6% and is now worth \$8600. How long ago did Abu buy the GPS system?
8. **BIOLOGY** In a laboratory, an organism grows from 100 to 250 in 8 hours. What is the hourly growth rate in the growth formula  $y = a(1 + r)^t$ ?

## 10-6

**Study Guide and Intervention** (continued)**Exponential Growth and Decay**

**Exponential Growth** Population increase and growth of bacteria colonies are examples of **exponential growth**. When a quantity increases by a fixed percent each time period, the amount of that quantity after  $t$  time periods is given by  $y = a(1 + r)^t$ , where  $a$  is the initial amount and  $r$  is the percent increase (or rate of growth) expressed as a decimal.

Another exponential growth model often used by scientists is  $y = ae^{kt}$ , where  $k$  is a constant.

**Example**

A computer engineer is hired for a salary of \$28,000. If she gets a 5% raise each year, after how many years will she be making \$50,000 or more?

Use the exponential growth model with  $a = 28,000$ ,  $y = 50,000$ , and  $r = 0.05$  and solve for  $t$ .

$$y = a(1 + r)^t$$

Exponential growth formula

$$50,000 = 28,000(1 + 0.05)^t$$

$$y = 50,000, a = 28,000, r = 0.05$$

$$\frac{50}{28} = (1.05)^t$$

Divide each side by 28,000.

$$\log\left(\frac{50}{28}\right) = \log(1.05)^t$$

Property of Equality of Logarithms

$$\log\left(\frac{50}{28}\right) = t \log 1.05$$

Power Property

$$t = \frac{\log\left(\frac{50}{28}\right)}{\log 1.05}$$

Divide each side by  $\log 1.05$ .

$$t \approx 11.9 \text{ years}$$

Use a calculator.

If raises are given annually, she will be making over \$50,000 in 12 years.

**Exercises**

- 1. BACTERIA GROWTH** A certain strain of bacteria grows from 40 to 326 in 120 minutes. Find  $k$  for the growth formula  $y = ae^{kt}$ , where  $t$  is in minutes.
- 2. INVESTMENT** Carl plans to invest \$500 at 8.25% interest, compounded continuously. How long will it take for his money to triple?
- 3. SCHOOL POPULATION** There are currently 850 students at the high school, which represents full capacity. The town plans an addition to house 400 more students. If the school population grows at 7.8% per year, in how many years will the new addition be full?
- 4. EXERCISE** Hugo begins a walking program by walking  $\frac{1}{2}$  mile per day for one week. Each week thereafter he increases his mileage by 10%. After how many weeks is he walking more than 5 miles per day?
- 5. VOCABULARY GROWTH** When Emily was 18 months old, she had a 10-word vocabulary. By the time she was 5 years old (60 months), her vocabulary was 2500 words. If her vocabulary increased at a constant percent per month, what was that increase?